Physics Letters B 745 (2015) 90–96

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Charged isotropic non-Abelian dyonic black branes

Yves Brihaye [a,*], Ruben Manvelyan [b], Eugen Radu [c], D.H. Tchrakian [d,e]

[a] Physique-Mathématique, Universite de Mons-Hainaut, Mons, Belgium
[b] Yerevan Physics Institute, Alikhanian Br. St. 2, 0036 Yerevan, Armenia
[c] Departamento de Física da Universidade de Aveiro and CIDMA Campus de Santiago, 3810-183 Aveiro, Portugal
[d] School of Theoretical Physics – DIAS, 10 Burlington Road, Dublin 4, Ireland
[e] Department of Computer Science, National University of Ireland Maynooth, Maynooth, Ireland

ARTICLE INFO

Article history:
Received 12 March 2015
Accepted 16 April 2015
Available online 20 April 2015
Editor: M. Cvetič

ABSTRACT

We construct black holes with a Ricci-flat horizon in Einstein–Yang–Mills theory with a negative cosmological constant, which approach asymptotically an AdS_d spacetime background (with $d \geq 4$). These solutions are isotropic, i.e. all space directions in a hypersurface of constant radial and time coordinates are equivalent, and possess both electric and magnetic fields. We find that the basic properties of the non-Abelian solutions are similar to those of the dyonic isotropic branes in Einstein–Maxwell theory (which, however, exist in even spacetime dimensions only). These black branes possess a nonzero magnetic field strength on the flat boundary metric, which leads to a divergent mass of these solutions, as defined in the usual way. However, a different picture is found for odd spacetime dimensions, where a non-Abelian Chern–Simons term can be incorporated in the action. This allows for black brane solutions with a magnetic field which vanishes asymptotically.

1. Introduction and motivation

In recent years there has been some interest in studying the AdS/CFT correspondence [1,2], in the presence of a background magnetic field. On the bulk side, this corresponds to solving the Einstein-gauge field system of equations, with suitable boundary conditions such that the AdS background is approached asymptotically, while the magnetic field does not trivialize. Several new classes of such solutions have been found in this way, most of them for the case of main interest of asymptotically AdS_5 configurations with Abelian fields. For example, the results in [3,4] revealed the existence of a variety of unexpected features of these solutions; here we mention only that their study is relevant for the issue of the third law of thermodynamics in the AdS/CFT context.

The investigation of the non-Abelian (nA) generalizations of these solutions is only in its beginning stages. Considering such configurations is a legitimate task, since the gauged supersymmetric models generically contain Yang–Mills fields (although usually only Abelian truncations are considered). To date, the only case investigated systematically corresponds to that in four ($d = 4$) spacetime dimensions (see [5] for a review of these solutions). The four-dimensional nA asymptotically-AdS (AAdS) solutions exhibit many new features which are absent for $\Lambda \geq 0$. For example, stable[1] solitons and black holes, possessing a global magnetic charge, are known to exist in a globally AdS_4 background even in the absence of a Higgs field [6,7]. However, the results in [8,9] show that these Einstein–Yang–Mills (EYM) black holes solutions have also generalizations with a nonspherical event horizon topology, in particular with a Ricci-flat horizon and a magnetic field which does not vanish asymptotically. They share many of the features of the spherical configurations in [6,7], in particular the existence of solutions stable against linear fluctuations. The only $d > 4$ nA AAdS solutions black holes studied more systematically so far are those possessing spherical event horizon topology [11–14], though some solutions with Ricci-flat horizon have been studied in [15,16].

In an unexpected development, the study of the $d = 4, 5$ EYM black brane solutions has led to the discovery of holographic superconductors and holographic superfluids, describing condensed phases of strongly coupled, planar, gauge theories [10]. Studying such solutions involves the construction of AAdS electrically charged black branes, which, below a critical temperature become unstable to forming YM hair. However, the magnetic field of these

* Corresponding author.
E-mail address: yves.brihaye@umons.ac.be (Y. Brihaye).

[1] The stability is against linear perturbations, and is not topological.

http://dx.doi.org/10.1016/j.physletb.2015.04.029
0370-2693/

configurations vanishes on the boundary, leading to a vanishing background magnetic field for the dual theory.

The main purpose of this work is to present an investigation of $d \geq 4$ AAdS isotropic black branes supporting both electric and magnetic nA fields. In contrast to previous studies in the literature, the magnetic fields of these solutions do not vanish on the boundary, which leads to a variety of interesting features. For example, we find that the mass of these asymptotically AdS solutions, as defined in the usual way, always diverges, while the solutions do not posses a regular extremal limit. In odd-dimensional spacetimes, when a Chern–Simons term is added to the total action, it is found that a special class of solutions exhibit a nontrivial magnetic field in the bulk while vanishing asymptotically.

2. The Einstein–Yang–Mills system

We consider the EYM theory in a d-dimensional spacetime, with a cosmological constant $\Lambda = -(d-2)(d-1)/(2L^2)$. The action is

$$I = \int_{\mathcal{M}} d^d x \sqrt{-g} \left(\frac{1}{16\pi G}(R - 2\Lambda) - \frac{1}{4} {*F} \wedge F \right) + S_{bndy}. \qquad (1)$$

The boundary terms S_{bndy} include the Gibbons–Hawking term [17] as well as the counterterms required for the on-shell action to be finite [18]. The Einstein and Yang–Mills equations derived from the above action are

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G\, T_{\mu\nu}, \qquad D_\mu F^{\mu\nu} = 0, \qquad (2)$$

where D_μ is the gauge derivative and the Yang–Mills stress-energy tensor

$$T_{\mu\nu} = \frac{1}{2} \left(F^{IJ}_{\mu\rho} F^{IJ}_{\nu\sigma} g^{\rho\sigma} - \frac{1}{4} g_{\mu\nu} F^{IJ}_{\rho\sigma} F^{IJ\rho\sigma} \right). \qquad (3)$$

We are interested in static Ricci-flat solutions which approach asymptotically a (planar) AdS$_d$ background. Also, to simplify the picture, we shall restrict our study to the following case: denoting the radial and time coordinate by r and t respectively and considering the hypersurfaces parametrized by x^i ($i = 1, \ldots, d-2$ and r, t fixed), we assume that all space directions in these hypersurfaces are equivalent. Thus the field strength and the metric are taken to be invariant under space translations and rotations in the planes (x^i, x^j); they are also time independent. Without any loss of generality, a line element with this property can be written in the form

$$ds^2 = g_{rr}(r) dr^2 + g_{\Sigma\Sigma}(r) d\Sigma^2_{d-2} + g_{tt}(r) dt^2, \qquad (4)$$

with $d\Sigma^2_{d-2} = (dx^1)^2 + \ldots + (dx^{d-2})^2$ the metric on the $(d-2)$-flat space.

The above symmetry requirements imply some restrictions on the choice of the gauge group. Restricting to $SO(n)$ YM fields, one finds that a YM ansatz leading to an isotropic energy–momentum tensor for *both* even and odd values of d is possible for $n \geq d+1$ only.[2]

In this work we shall consider an $SO(d+1)$ gauge group, with $d(d-1)/2$ $SO(d+1)$ nA gauge fields represented by the 1-form potential A^{IJ} antisymmetric in I and J (with $I, J = 1, \ldots, d+1$) and $F^{IJ} = dA^{IJ} + \frac{1}{\hat{g}} A^{IK} \wedge A^{KJ}$, with $\hat{g}$ the Yang–Mills coupling. Also, to simplify the relations, it is convenient to define

$$\alpha^2 = \frac{4\pi G}{\hat{g}^2}. \qquad (5)$$

3. Embedded Abelian solutions

Before proceeding to the non-Abelian case, it is instructive to consider the dyonic black branes in Einstein–Maxwell theory, (i.e. the gauge fields taking their values in the $U(1)$ subgroup of $SO(d+1)$). A gauge field ansatz compatible with the symmetries of the line-element (4) can be constructed for an even number of spacetime dimensions only, $d = 2n+2$ and reads[3]

$$\begin{aligned}
&A^{IJ}_1 = \frac{w_0^2}{\hat{g}} x^2 \delta^I_{[d} \delta^J_{d+1]}, \quad A^{IJ}_2 = -\frac{w_0^2}{\hat{g}} x^1 \delta^I_{[d} \delta^J_{d+1]}, \\
&\ldots, \\
&A^{IJ}_{2n-1} = \frac{w_0^2}{\hat{g}} x^{2n} \delta^I_{[d} \delta^J_{d+1]}, \quad A^{IJ}_{2n} = -\frac{w_0^2}{\hat{g}} x^{2n-1} \delta^I_{[d} \delta^J_{d+1]}, \\
&A^{IJ}_r = 0, \quad A^{IJ}_t = \frac{V(r)}{\hat{g}} \delta^I_{[d} \delta^J_{d+1]},
\end{aligned} \qquad (6)$$

with w_0 an arbitrary parameter which fixes the magnetic field in a two plane, $F^{IJ}_{21} = \ldots = F^{IJ}_{2n\,2n-1} = \frac{2w_0^2}{\hat{g}} \delta^I_{[d} \delta^J_{d+1]}$. Choosing a metric gauge with $g_{\Sigma\Sigma} = r^2$, one finds[4] a black brane solution with $1/g_{rr} = -g_{tt} = N(r)$, where

$$\begin{aligned}
N(r) = \frac{r^2}{L^2} - \frac{M_0}{r^{d-3}} + \frac{2}{(d-3)(d-2)} \frac{\alpha^2 Q^2}{r^{2(d-3)}} \\
- \frac{4}{(d-5)} \frac{\alpha^2 w_0^4}{r^2},
\end{aligned} \qquad (7)$$

and

$$V(r) = V_0 - \frac{Q}{(d-3) r^{d-3}}, \qquad (8)$$

with V_0 a constant which is fixed by requiring that the electric potential vanish at the horizon. Apart from w_0, this solutions possesses two more parameters: M_0 and Q, which fixes the mass and the electric charge densities, respectively.

This black brane possesses an horizon at $r = r_H > 0$, where $N(r_H) = 0$ (and $N'(r_H) \geq 0$). The Hawking temperature T_H, the event horizon area density A_H, the chemical potential Φ and the electric charge density Q_e of this solution are

$$\begin{aligned}
&T_H = \frac{1}{4\pi} \left((d-1) \frac{r_H}{L^2} - \frac{2\alpha^2}{r_H} \left(\frac{2w_0^4}{r_H^2} + \frac{1}{(d-2)} \frac{Q^2}{r_H^{2(d-3)}} \right) \right), \\
&A_H = r_H^{d-2}, \\
&\Phi = \frac{1}{d-3} \frac{Q}{r_H^{d-3}}, \quad Q_e = \frac{\alpha^2}{4\pi} Q.
\end{aligned} \qquad (9)$$

One can easily verify that the total mass of the solutions, as defined according to the counterterm prescription in [18], diverges for any (even) $d > 4$ due to the slow decay of the magnetic fields, despite the fact that the spacetime is still AAdS. A finite mass density results when a boundary term

[2] Note that, for even values of d, one can consider instead a gauge group $SO(d-1)$, which leads to isotropic EYM branes. A study of this case has been proposed in [15] (Ansatz I there). However, the properties of those solutions are rather different to the case of interest here.

[3] The ansatz (6), (4) can be extended to the case of odd d by adding a number of codimensions y^μ, with $A^{IJ}_\mu = 0$; however, this leads to anisotropic configurations.

[4] A version of this solution has been considered in a more general context in [24]. Also, its purely magnetic limit, $Q = 0$, has been discussed in [3].

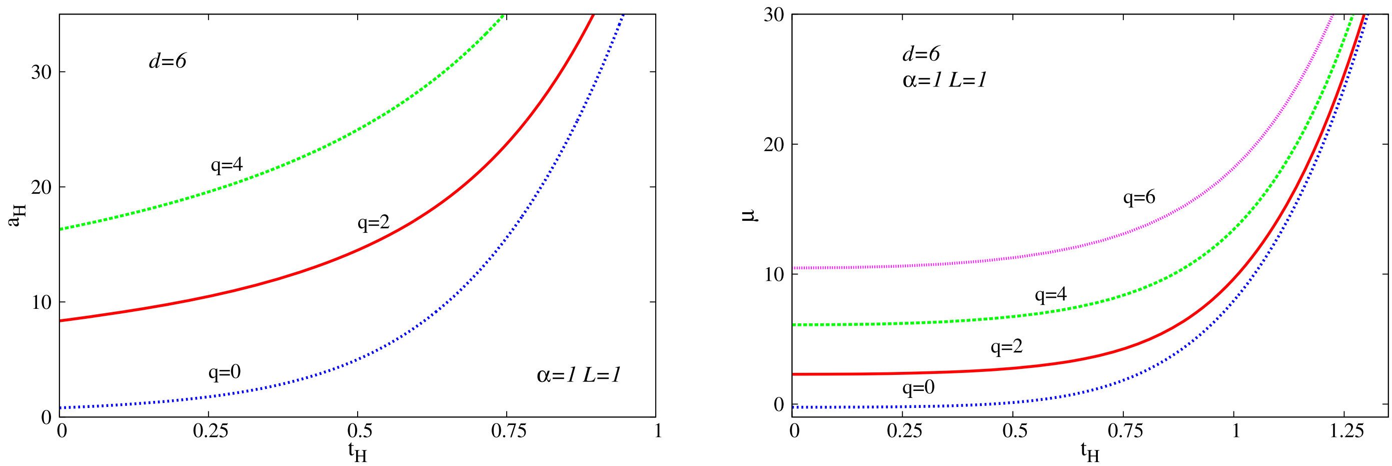

Fig. 1. The reduced area a_H and mass μ are shown as a function of reduced temperature t_H for $d=6$ isotropic black branes in Einstein–Maxwell theory. Here and in Fig. 3 the quantities are scaled with respect to the magnetic field on the boundary.

$$I_{ct}^{(YM)} = -\frac{1}{d-5}\int_{\partial\mathcal{M}} d^{d-1}x\sqrt{-h}\frac{L}{4}\,F^{IJ}_{ab}F^{IJ\,ab}\,, \tag{10}$$

is included in (1), with h_{ab} the boundary metric and F^{IJ}_{ab} the gauge field on the boundary. Then the boundary stress tensor $T_{ab} = \frac{2}{\sqrt{-h}}\frac{\delta I}{\delta h^{ab}}$ acquires a supplementary contribution from (10), which leads to finite mass density[5]

$$M = \frac{(d-2)}{16\pi G}M_0. \tag{11}$$

Note that this relation holds also for the simplest case $d=4$, in which case no matter counterterm is required.

One can see that the quantities (9), (11) verify the first law of thermodynamics (with a constant background magnetic field)

$$dM = \frac{1}{4G}T_H dA_H + \frac{1}{G}\Phi dQ_e. \tag{12}$$

In discussing the properties of these solutions (and their non-Abelian generalizations), it is convenient to work with quantities scaled with respect to the magnetic field in a two plane as fixed by the parameter w_0:

$$a_H = \frac{A_H}{w_0^{d-2}},\quad t_H = \frac{T_H}{w_0},\quad \mu = \frac{GM}{w_0^{d-1}},\quad q = \frac{Q_e}{w_0^{d-2}}. \tag{13}$$

As seen in Fig. 1, the properties of the solutions with a background magnetic field are not really sensitive to the presence of an electric charge since the constant-q curves preserve the $q=0$ shape, which is approached asymptotically for large t_H. These dyonic black branes possess a regular extremal limit $T_H=0$, with an $AdS_2\times R^{d-2}$ near horizon geometry. An interesting feature is that the total mass of the $d>4$ solutions is allowed to take negative values. This can easily be seen in the extremal case, a limit which is approached for $Q = \frac{r_H^{d-2}}{\sqrt{2}\alpha L}\sqrt{(d-2)(d-1)}\sqrt{1-\frac{4\alpha^2L^2w_0^4}{(d-1)r_H^4}}$. The extremal solutions have a mass

$$M = \frac{(d-2)^2(d-1)}{8(d-3)\pi L^2}\left(1-\frac{4(d-4)\alpha^2L^2w_0^4}{(d-5)(d-2)(d-1)r_H^4}\right), \tag{14}$$

which becomes negative for $\frac{(d-5)(d-2)}{4(d-4)\alpha^2L^2} < \frac{w_0^4}{r_H^4} \le \frac{(d-1)}{4\alpha^2L^2}$.

4. Non-Abelian solutions

A simple nA ansatz leading to an isotropic line element can be constructed for any $d\ge 4$, in terms of a magnetic potential, $w(r)$ and an electric one $V(r)$

$$A_i^{IJ} = \frac{w(r)}{\hat{g}}\delta^I_{[i}\delta^J_{d-1]},\quad A_r^{IJ}=0,\quad A_t^{IJ} = \frac{V(r)}{\hat{g}}\delta^I_{[d}\delta^J_{d+1]}. \tag{15}$$

Unfortunately, no AAdS exact solutions of the EYM equations seems to exist in this case. However, the system possesses a simple globally regular Lifshitz-type configuration with

$$ds^2 = c_1\frac{dr^2}{r^2} + c_2 r^2 d\Sigma^2_{d-2} - r^{2z}dt^2,$$
$$w(r) = u_0 r,\quad V(r) = 0, \tag{16}$$

where

$$c_1 = \frac{4\alpha^2}{(d-2)p^2},\quad c_2 = \frac{2\alpha^2\left(2(d-3)-(d-2)p^2\right)}{(d-2)^2p^2}u_0^2,$$
$$z = \frac{(d-3)((d-2)p^2+2)}{2(d-3)-(d-2)p^2} > 1, \tag{17}$$

here $u_0\neq 0$ is an arbitrary constant while p is a parameter related to the cosmological constant by

$$\Lambda = -\frac{(d-2)p^2}{2\alpha^2((d-2)q^2-2(d-3))}\times\Big((d-2)p^4+(d-2)(d-3)(d(d-6)+4)p^2 + 4(d-3)^2(d-1)\Big),$$

and obeying the condition $p<\sqrt{2(d-3)/(d-2)}$. The solution (16) possesses the Lifshitz scaling symmetry $t\to\lambda^z t$, $x^i\to\lambda x^i$, $r\to r/\lambda$ and generalizes the $d=4$ EYM solution of Ref. [19] to higher dimensions. As discussed there, in this case the field equations possess black brane solutions with a regular horizon approaching the background (16) as $r\to\infty$. We expect the existence of similar black brane solutions for $d>4$ as well.

Returning to the case of solutions with AdS asymptotics, it turns out convenient for the numerical construction to choose a metric ansatz of the form

[5] As usual in this context, the mass is the charge associated with the time-translation symmetry of the boundary metric.

$$g_{rr}=\frac{1}{N(r)},\quad g_{\Sigma\Sigma}=r^2,$$
$$g_{tt}=-N(r)\sigma^2(r),\quad \text{with}\quad N(r)=\frac{r^2}{L^2}-\frac{m(r)}{r^{d-3}}. \tag{18}$$

Inserting this ansatz into the Einstein and Yang–Mills equations yields four equations of motion[6] for $m(r)$, $\sigma(r)$, $w(r)$ and $V(r)$ (a prime denotes $\frac{d}{dr}$):

$$m'=2\alpha^2 r^{d-4}\left(\frac{1}{d-2}\frac{r^2V'^2}{\sigma^2}+Nw'^2+\frac{(d-3)}{2r^2}w^4\right),$$
$$\sigma'=\frac{2\alpha^2}{r}\sigma w'^2,$$
$$w''+\left(\frac{d-4}{r}+\frac{N'}{N}+\frac{\sigma'}{\sigma}\right)w'-(d-3)\frac{w^3}{r^2N}=0,$$
$$V''+\left(\frac{d-2}{r}-\frac{\sigma'}{\sigma}\right)V'=0 \tag{19}$$

The last equation above implies the existence of the first integral

$$V'=\sigma\frac{Q}{r^{d-2}}, \tag{20}$$

with Q a constant fixing the electric charge of the solutions.

The equations of motion (19) are invariant under three scaling transformations (invariant quantities are not shown):

$$\begin{aligned}
&\text{(I)}\quad \sigma\to\lambda\sigma,\qquad V\to\lambda V,\\
&\text{(II)}\quad r\to\lambda r,\quad m\to\lambda^{d-1}m,\quad w\to\lambda w,\quad V\to\lambda V,\\
&\text{(III)}\quad r\to\lambda r,\quad m\to\lambda^{d-3}m,\quad L\to\lambda L,\quad V\to\frac{V}{\lambda},\quad \alpha\to\lambda\alpha,
\end{aligned} \tag{21}$$

where λ represents the positive (real) scaling parameter. Using (I), we set the boundary values of the metric function $\sigma(r)$ to one, so that the metric will be asymptotically (locally) AdS. We are free to use (II) to set the asymptotic value of the magnetic potential $w(r)$ to an arbitrary (non-vanishing) value (equivalently, one can use this symmetry to fix the value of the electric charge or the horizon radius of the solution, say r_H). Finally, the symmetry (III) can be used to fix the value of the AdS radius L or the value of the coupling constant α; for most of the work in this paper we set $\alpha=1$ (thus we treat L as an input parameter).

Denoting the position of the horizon of the black brane solutions by r_H, we have to impose $N(r_H)=0$ (and $N'(r_H)\geq 0$) while the other metric functions stay strictly positive. A nonextremal solution has the following expression near the event horizon:

$$m(r)=\frac{r_H^{d-1}}{2L^2}+m'(r_H)(r-r_H)+O(r-r_H)^2,$$
$$\sigma(r)=\sigma_H+\sigma'(r_H)(r-r_H)+O(r-r_H)^2,$$
$$w(r)=w_H+w'(r_H)(r-r_H)+O(r-r_H)^2,$$
$$V(r)=V'(r_H)(r-r_H)+O(r-r_H)^2, \tag{22}$$

where

$$m'(r_H)=\frac{2\alpha^2Q^2}{(d-2)r_H^{d-2}}+\alpha^2(d-3)\frac{w_H^4}{r_H^{d-6}},$$
$$w'(r_H)=\frac{(d-3)L^2w_H^3}{r_H^3\left(d-1-\frac{\alpha^2L^2}{r_H^4}\left(\frac{2Q^2}{(d-2)r_H^{2(d-4)}}+(d-3)w_H^4\right)\right)},$$
$$V'(r_H)=\frac{Q}{r_H^{d-3}},$$
$$\sigma'(r_h)=-\frac{2\alpha^2(d-3)^2L^4\sigma_Hw_H^6}{r_H^7\left(d-1-\frac{\alpha^2L^2}{r_H^4}\left(\frac{2Q^2}{(d-2)r_H^{2(d-4)}}+(d-3)w_H^4\right)\right)}, \tag{23}$$

with w_H and σ_H arbitrary constants.

The AdS boundary is reached as $r\to\infty$. We are interested in configurations with $w(r)\to w_0\neq 0$, such that the magnetic field on the boundary is nonvanishing, $F_{ij}^{IJ}=-\frac{1}{g}w_0^2\delta_{[i}{}^I\delta_{j]}{}^J$. A straightforward but cumbersome computation leads to the following general asymptotic expression of the solutions as $r\to\infty$ (note the presence of log terms for an odd value of the spacetime dimension):

$$\begin{aligned}
m(r)&=M_0+\frac{\alpha^2L^{d-5}w_0^{d-1}}{d-2}\\
&\quad\times\log\left(\frac{r}{L}\right)\left(6\delta_{d,5}-40\delta_{d,7}+\frac{567}{4}\delta_{d,9}+\dots\right)\\
&\quad+\frac{\alpha^2(d-3)}{(d-5)}w_0^4r^{d-5}\Theta(d-6)\\
&\quad-\frac{2\alpha^2L^2(d-3)^2(d-6)}{(d-5)^2(d-7)}w_0^6r^{d-7}\Theta(d-8)+\dots,\\
\sigma(r)&=1-\frac{4}{3}\alpha^2w_0^6\log^2\left(\frac{r}{L}\right)\frac{L^4}{r^6}\delta_{d,5}\\
&\quad-\frac{\alpha^2(d-3)^2L^4w_0^6}{3(d-5)^2r^6}\Theta(d-6)+\dots,\\
w(r)&=w_0+\frac{J}{r^{d-3}}+\frac{w_0^{d-2}L^{d-3}}{r^{d-3}}\\
&\quad\times\log\left(\frac{r}{L}\right)\left(-\delta_{d,5}+3\delta_{d,7}-\frac{27}{4}\delta_{d,9}+\dots\right)\\
&\quad-\frac{d-3}{d-5}\frac{w_0^3L^2}{2r^2}\Theta(d-6)\\
&\quad+\frac{3(d-3)^2}{8(d-5)(d-7)}\frac{w_0^5L^4}{r^4}\Theta(d-8)+\dots,\\
V(r)&=V_0-\frac{Q}{r^{d-3}}+\dots,
\end{aligned} \tag{24}$$

The series truncates for any fixed dimension, with new terms entering at every new even value of d, as denoted by the step-function ($\Theta(x)=1$ provided $x\geq 0$, and vanishes otherwise). The constants w_0, M_0, V_0 and J in the above expressions are free parameters which are fixed by numerics.

As in the Abelian case, we expect the parameter M_0 to encode the mass density of the solutions, which is still given by (11). However, a rigorous proof of this statement is rather difficult, due to the complicated asymptotic behavior of the metric functions. For $d=5$, a regularized boundary energy–momentum tensor and mass are found by including in (1) the following matter counterterm

$$I_{ct}^{YM}=-\log\left(\frac{r}{L}\right)\int_{\partial\mathcal{M}}d^4x\sqrt{-h}\frac{L}{4}F_{ab}^{IJ}F^{IJ\,ab}. \tag{25}$$

[6] One extra equation containing the second derivatives of the metric functions $m(r)$, $\sigma(r)$ is also found. However, one can show that this constraint equation is implicitly satisfied for the set of boundary conditions chosen.

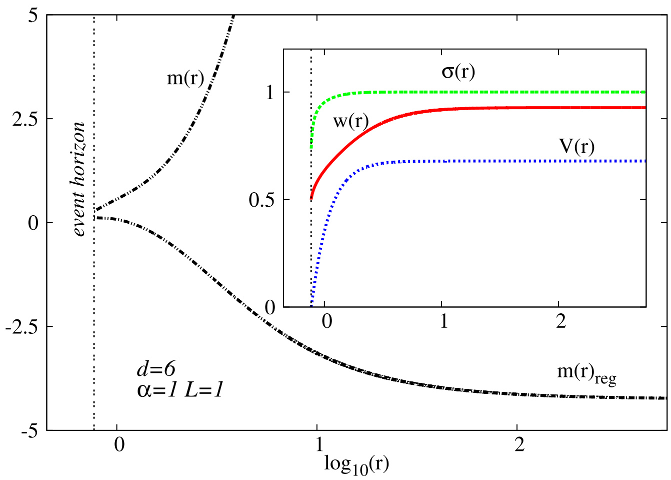

Fig. 2. The profiles of a typical $d = 6$ Einstein–Yang–Mills isotropic black brane solution are shown as a functions of the radial coordinate r.

We have found that the boundary counterterm (10) regularizes also the mass of the $d = 6$ solutions. In both cases, this results in the expression (11) for the mass density of the black branes. (Note that (11) holds also for $d = 4$, in which case no matter counterterm is necessary.) However, the above simple counterterm fails to regularize all divergencies in the expression of M for $d > 6$. Thus a more general matter counterterm than (10) is required in the $d > 6$ case. We find that for any $d \geq 4$, the mass of the solutions computed by integrating the first law equation (12), coincides with the relation (11) with good accuracy.

Other quantities which enter the thermodynamics of the solutions are given by

$$A_H = r_H^{d-3}, \quad T_H = \frac{1}{4\pi} N'(r_H)\sigma(r_H),$$
$$\Phi = V_0, \quad Q_e = \frac{\alpha^2}{4\pi} Q. \tag{26}$$

Solutions interpolating between the near horizon expansion (22) and the far field asymptotics (24) are constructed numerically, using a standard Runge–Kutta ordinary differential equations solver. In this approach we evaluate the initial conditions at $r = r_H + 10^{-5}$, for global tolerance 10^{-14}, adjusting for shooting parameters and integrating towards $r \to \infty$ (thus we have restricted our study to the region outside the event horizon). The equations were integrated for all values of d between four and ten; thus similar solutions are expected to exist for any value of d.

For a given d, we have considered a range of values for (r_H, w_H, Q), the parameters σ_H and M_0, V_0, J resulting from the numerical output. Since Eqs. (19) are invariant under the transformation $w \to -w$, only values of $w_H > 0$ are considered. Also, we have studied mainly the case where the AdS length scale is set to one, $L = 1$. The profile of a typical $d = 6$ non-Abelian solution is shown in Fig. 2. (There we have displayed also the mass function density $m(r)_{reg}$ regularized via the counterterm (10).)

We have found that the nA solutions share most of the basic properties of the Einstein–Maxwell configurations discussed above. In particular, the presence of an electric charge does not change qualitatively the general picture. Also, a number of basic features of these black holes are similar to those of the known $d = 4$ (purely magnetic) configurations in [8]. This can be understood by noticing that, for our choice of the ansatz, the magnetic and electric potentials interact only via the spacetime geometry. As a result, these black branes can be thought of as nonlinear superpositions of purely electric Reissner–Nordström–AdS solutions (i.e. the limit $w_0 = 0$ in (6), (7)) and purely magnetic nA configurations[7] with $V(r) = 0$. This can be seen in Fig. 3, where we plot the event horizon area and the mass of $d = 5$ solutions, for several (fixed) values of the electric charge; note that in that plot the quantities are normalized w.r.t. to the magnetic field on the boundary, as defined by (13), which remain invariant under the transformation (ii) in (21). One can easily see that the corresponding $q = 0$ curves are generic. Also, as in the Abelian case, we have noticed the existence of $d > 5$ solutions with a negative total mass, $M_0 < 0$, see Fig. 2 (solutions with $M_0 = 0$ do also exist).

However, the limiting behavior of the EYM solution is very different from the Abelian case, the limit $T_H \to 0$ being singular this time. This can be understood by noticing that the non-linearity of the YM equation for the magnetic potential implies the absence of a $AdS_2 \times R^{d-2}$ near horizon geometry as a solution of the field equations.

5. Non-Abelian black branes in odd dimensions with a Chern–Simons term

In odd spacetime dimensions, the usual gauge field action can be augmented with a Chern–Simons (CS) term. Such a term typically enters the action of gauged supergravities, the case of $\mathcal{N} = 8$, $d = 5$ model with a gauge group $SO(6)$, being perhaps the most interesting.[8]

The expression of the CS Lagrangean for the case $d = 5$ discussed in what follows, is[9]

$$L_{CS} = \kappa \epsilon_{I_1 \cdots I_6} \Big(F^{I_1 I_2} \wedge F^{I_3 I_4} \wedge A^{I_5 I_6}$$
$$- \hat{g} F^{I_1 I_2} \wedge A^{I_3 I_4} \wedge A^{I_5 J} \wedge A^{J I_6}$$
$$+ \frac{2}{5} \hat{g}^2 A^{I_1 I_2} \wedge A^{I_3 J} \wedge A^{J I_4} \wedge A^{I_5 K} \wedge A^{K I_6} \Big), \tag{27}$$

with κ an arbitrary parameter, the CS coupling constant.[10]

One can easily show that the Abelian configuration (7) still remains a solution in the presence of a CS term[11]; however, the situation is different for non-Abelian fields. These solutions can be studied within the same ansatz (15), (18); the equations for metric functions $m(r)$, $\sigma(r)$ are still valid, since the CS term does not contribute to the energy–momentum tensor, while the equations for the gauge potentials contain new terms encoding a direct interaction between magnetic and electric potentials:

$$w'' + \left(\frac{d-4}{r} + \frac{N'}{N} + \frac{\sigma'}{\sigma} \right) w' - (d-3) \frac{w^3}{r^2 N}$$
$$- \kappa \frac{(d^2-1)}{(d-2)} \frac{w^{d-3} V'}{N \sigma r^{d-4}} = 0, \tag{28}$$

[7] One interesting feature is the absence of solutions with nodes of the magnetic potential. This can be analytically proven by integrating the equation for w, $(N\sigma r^{d-4} w')' = (d-3) w^3 \sigma r^{d-6}$, between r_H and some r; obtaining $w' > 0$ for every $r > r_H$. In a similar way, one can prove that the metric function $\sigma(r)$ monotonically increases towards its asymptotic value.

[8] Note that a simple EYMCS theory does not seem to correspond to a consistent truncation of any supergravity model. However, we expect that the basic properties of our solutions would hold also in that case (see Ref. [23] for a study of nA in of the $\mathcal{N} = 4^+$, $d = 5$ gauged supergravity model, which contains also a CS term).

[9] The explicit expression of the CS Lagrangean for $d = 7$, 9 can be found e.g. in Ref. [20].

[10] The value of κ is fixed in supersymmetric theories, but in this work we treat κ as a free input parameter.

[11] Note the situation changes for *anisotropic* dyonic Abelian black branes, in which case the inclusion of a U(1) CS term leads to variety of new interesting properties, see e.g. [4].

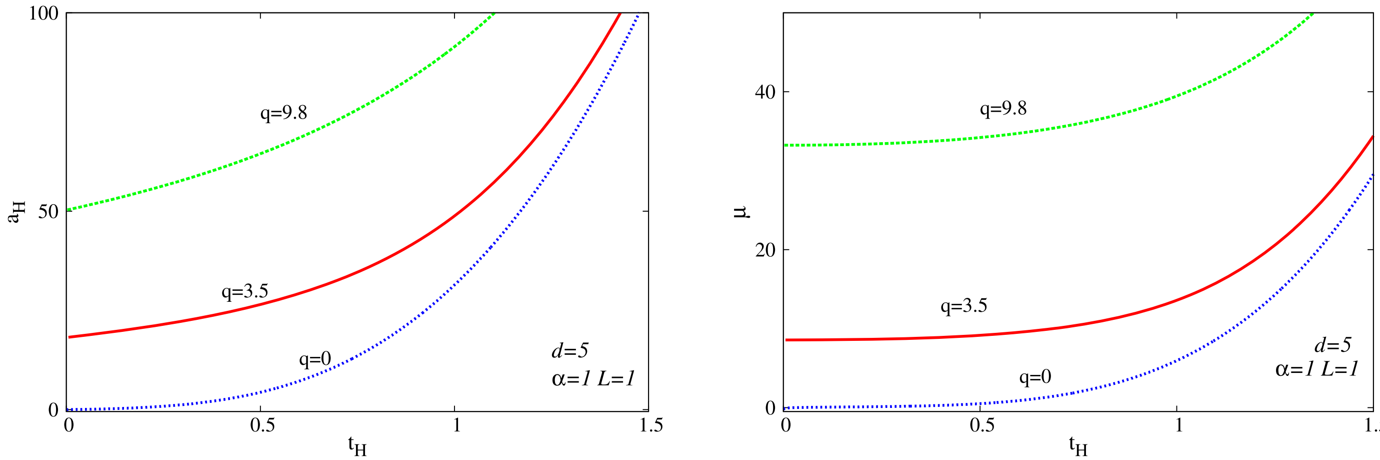

Fig. 3. The reduced area a_H and mass μ are shown as functions of the reduced temperature t_H for $d=5$ isotropic black branes in Einstein–Yang–Mills theory.

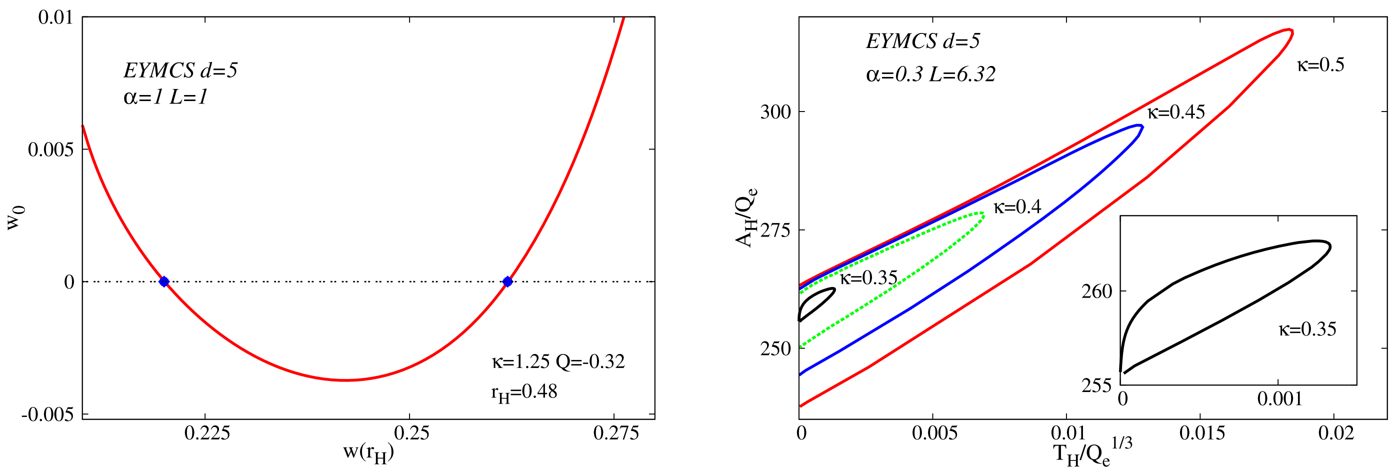

Fig. 4. *Left:* The asymptotic value of the magnetic gauge potential w_0 is shown as a function of its value at the event horizon $w(r_H)$ for $d=5$ solutions in Einstein–Yang–Mills–Chern–Simons theory. *Right:* The scaled horizon area A_H is shown – for several values of κ – as a function of the scaled temperature T_H for $d=5$ Einstein–Yang–Mills–Chern–Simons solutions with a vanishing magnetic field on the boundary.

$$V' = \frac{\sigma}{r^{d-2}}(Q + \kappa \frac{(d^2-1)}{(d-2)} w^{d-2}), \tag{29}$$

with Q an integration constant.

By using a similar approach to that described above, we have studied families of $d=5$ solutions of the EYM–CS model in a systematic way.[12]

The EYM–CS solutions possess a near horizon expansion similar to (22), while their leading order terms in the far field expression is still given by (24), κ entering through the lower terms only.

We have found that all basic properties of the solutions without a CS term are retained in this case. However, some new features occurs as well, the most interesting being the existence of configurations with $w(\infty)=0$.

For a special set of event horizon data, one finds solutions with vanishing magnetic field on the AdS boundary (although $w(r)$ is nonzero in the bulk). From (24), this implies that in this case, as $r \to \infty$ the mass function $m(r)$ approaches a finite value. This feature is illustrated in Fig. 4 (left), where we plot w_0, the asymptotic value of the magnetic gauge potential w, as a function of the value of the magnetic potential on the horizon for fixed values of κ, Q, r_H and L (the special value of $w(r_H)$ which correspond to $w(\infty)=0$) are marked with dots).

Naively, this resembles the solutions describing holographic p-wave superconductors and superfluids which have been extensively studied in recent years, starting with the seminal work [10]. However, the overall picture is rather different for the EYMCS solutions obtained here. First, in contrast to the EYM solutions of Refs. [10,15,16], these configurations do not emerge as a perturbation of the RN–AdS Abelian solution.[13] Second, the general pattern of the EYMCS black branes with a vanishing magnetic field on the boundary is different from the one corresponding to nA configurations without a CS term. For example, as seen in Fig. 4 (right), the EYMCS black branes with given (α, κ) form two branches of solutions. These branches extend up to a maximal value of the Hawking temperature and horizon area, where they join (note that the quantities plotted are scale invariant under (ii) in (21) by an appropriate combination with the electric charge).

Interestingly (and in strong contrast to the pure EYM case discussed above), the limit $T_H \to 0$ corresponds to extremal solutions

[12] We expect the properties of the five-dimensional solutions to be generic. Indeed, this is supported by the preliminary results we have found for EYMCS solutions in $d=7$ spacetime dimensions.

[13] That is, when treating $w(r)$ as a small perturbation around the electrically charged RN–AdS black brane, one finds that the solution of the YM–CS linearized equation (28) possesses an essential logarithmic singularity at the horizon. However, we have verified that the EYMCS hairy solutions with $w(\infty)=0$ are thermodynamically favoured over the RN-AdS Abelian configurations, *i.e.* they minimize the free energy for the same Q, T_H.

with a regular horizon. Such configurations possess an $AdS_2 \times R^3$ near horizon geometry, with[14]

$$ds^2 = v_1(\frac{dr^2}{r^2} - r^2 dt^2) + v_2 d\Sigma_3^2, \text{ and}$$
$$w(r) = w_0, \;\; V(r) = qr, \tag{30}$$

(where the redefinition $r - r_H \to r$ is implicitly assumed) and

$$v_1 = \frac{2}{3}\left(\frac{8}{L^2} - \frac{\alpha^2 w_0^2}{16\kappa^2(Q + 8\kappa w_0^3)^2}\right)^{-1},$$
$$v_2 = -\frac{4\kappa(Q + 8\kappa w_0^3)}{w_0}, \text{ and } q = \frac{v_1(Q + 8\kappa w_0^3)}{v_2^{3/2}}. \tag{31}$$

Given κ, α and L, this configuration possesses one single free parameter, the constants Q, w_0 satisfying the algebraic equation

$$512\kappa^2(Q + 8\kappa w_0^3)^2 + \alpha^2 L^2 w_0^3(Q - 4\kappa w_0^3) = 0. \tag{32}$$

We note that the overall picture possesses a nontrivial dependence on the value of the CS coupling constant, with the existence of a minimal value of κ allowing for a vanishing magnetic field on the boundary. We hope to return elsewhere with a systematic study of the EYMCS configurations, in a more general context.

6. Conclusions

In this work we have constructed isotropic black branes in an AdS_d background possessing both electric and magnetic $SO(d+1)$ non-Abelian fields. The solutions were obtained by using a combination of analytical and numerical methods. Several basic properties of these solutions in $d > 4$ can hardly be anticipated from the study of their four-dimensional counterparts. For example, the magnetic field of the EYM solutions does not vanish asymptotically. As a result their mass – defined in the usual way – always diverges. However, solutions with a finite mass exist – in odd spacetime dimensions – when supplementing the action by a Chern–Simons term.

There are various possible natural extensions of this work. Perhaps the most interesting one would be to study the transport properties of these solutions. Investigation of the thermodynamics of the black branes is another important problem. Here we mention only that the heat capacity is always positive for the EYM black holes in a canonical ensemble. As a result, these configurations are always thermodynamically locally stable, a feature shared with the vacuum solutions. Finally, note that the YM ansatz used in this work is not the most general one leading to an isotropic black brane; for instance the components of the connection (15) take their values in the algebra of $SO(d-1) \times U(1)$ and not in the full algebra of $SO(d+1)$. The fully $SO(d+1)$ YM ansatz can be written in terms of two magnetic potentials and two electric potentials, and is expected to lead to a more complicated picture.[15]

Acknowledgements

We acknowledge M. Ortaggio for bringing Ref. [24] to our attention. The work of Y.B. was supported in part by an ARC contract AUWB-2010/15-UMONS-1. E.R. gratefully acknowledges support from the FCT-IF programme and CIDMA strategic project UID/MAT/04106/2013.

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[14] This configuration can be generalized for any (odd) $d \geq 5$; however, the relations are much more complicated in the general case.

[15] The $d = 5$ EYMCS counterpart of these configurations with a spherical horizon topology have been studied in [14,21]; see also [22,20] for the $\Lambda = 0$ limit of these solutions.